



Intertemporal coordination with delay options

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Abstract

This paper studies equilibrium selection in intertemporal coordination problems with delay options. The risk-dominant action of the underlying one-shot game is selected when frictions are arbitrarily small. Larger frictions introduce real option effects in the model and inhibit coordination.

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1. Introduction

Intertemporal coordination problems, where an agent's payoff depends on the future behavior of other agents, are frequent in economics. In the R&D industry, the benefit of current innovations depends on the emergence of complementary future innovations. In asset markets, the benefit of holding a particular asset depends on its fundamental value but also on its liquidity, i.e., the facility with which the asset may be accepted in trade in the near future. Of particular importance in such problems is how the possibility of delaying a costly action (e.g., an innovation, the acquisition of an asset) may impact coordination. In this paper we examine this issue in

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the context of an OLG model, a prototypical environment in which intertemporal coordination matters.

In the standard OLG model, an agent that exerts effort when young is entitled to a benefit when old only if the young agent of the following generation exerts effort as well. By assumption, there are no delays, i.e., a young agent only gets one chance to exert effort, and an old agent only gets one chance to receive the benefit. If one abstracts from the coordination problem and let agents believe future agents will always exert effort, there exists an equilibrium in which every young agent exerts effort. This equilibrium is immune to the introduction of delay options since this option is never exercised.

We augment the standard OLG model in two ways. First, we let the economy experience different states over time, which evolve according to a random walk, and the cost of exerting effort is an increasing function of the current state. In a large region of states, the region of interest, the present value of receiving the benefit in the next period is larger than the cost of exerting effort in the current period. There exist though faraway states in which it is strictly dominant to exert effort and faraway states in which it is strictly dominant not to do so. Second, we introduce delay options. In every period, a young agent (relabelled active agent) chooses between exerting and delaying effort to an old agent (relabelled passive agent). If she chooses effort, she incurs a sunk cost, which depends on the current state, and becomes a passive agent, while the passive agent receives the benefit and is replaced by a newly born active agent. If she chooses to delay effort, nothing changes and both active and passive agents move to the next period.

We first prove that there exists a unique equilibrium characterized by a threshold: agents exert effort if and only if the current state is at the left of the threshold. We then show that, in the case of vanishing shocks, delay options do not matter: effort is exerted if and only if effort is also exerted in the hypothetical scenario where the agent only gets one chance to exert effort and only one chance to receive the benefit from her future partner. A corollary of this result is that if the state evolves according to a symmetric distribution, effort is exerted if and only if it is the risk dominant action in the corresponding one-shot game between the current active agent and her future partner. In fact, in the absence of delays, agents are essentially playing a one-shot game: they can neither delay effort nor wait longer to receive the benefit from their future partner's effort. If the agent is currently at the equilibrium threshold, his future partner will choose effort with probability half. This implies that the agent will exert effort if and only if it is the risk dominant action in the corresponding one-shot game.¹

The result that delay options do not matter for equilibrium selection in the limit of vanishing shocks is quite surprising. Indeed, the possibility of delays substantially complicates the problem of equilibrium selection: an agent knows she can wait for many periods before getting the benefit of her effort, and also that she will have further opportunities for effort. It turns out that, if an agent is at the equilibrium threshold, the extra benefits from immediate effort and the possible gains from exerting effort later exactly offset each other. Thus although standard OLG models do not allow for the possibility of delays, this restriction has no effect on coordination in the case of vanishing shocks.

The selection of the risk dominant-action is a result that often emerges in the literature of coordination games. In two-by-two coordination games with incomplete information (global games),

¹ The risk dominance criterion implies that the agent chooses the most profitable action under maximum uncertainty about the others' actions. In a two-by-two game, that means assuming the other agent will choose either action with probability half.

Carlsson and Van Damme [7] showed that the risk-dominant equilibrium is selected. The argument has then been extended to games with a continuum of agents (Morris and Shin [20]), a continuum of actions (Guimaraes and Morris [13]) and large heterogeneous populations (Sakovics and Steiner [23]). The risk dominant equilibrium of the corresponding one-shot game also arises in an evolutionary context (Kandori, Mailath and Rob [15]) and in a repeated-game setting from stability arguments (Matsui and Matsuyama [19]). Last, Burdzy, Frankel and Pauzner [6] study dynamic games with frictions that prevents agents from changing strategy at every moment and the risk-dominant equilibrium of the one-shot game is selected as frictions vanish.

When we move away from the case of vanishing shocks, a positive relation between the variance of the random process and the cut-off state arises. Delay options thus hinder coordination when the variance of the random process is bounded away from zero. This captures a real option effect (e.g., Bloom [4]): an increase in uncertainty (variance) leads to a reduction in the likelihood that effort will be exerted because it increases the value of postponing effort.

This paper is related to the literature of dynamic coordination games with frictions.² Unlike these papers, our focus is on intertemporal coordination with the option to delay. The paper is also related to the literature on dynamic global games.³ In particular, Dasgupta [9], Kovac and Steiner [16] and Mathevet and Steiner [18] consider environments where the possibility of learning might generate incentives to delay a risky action and find that in general, delay options affect the equilibrium.⁴ Here in contrast, there is complete information about the state of the economy – which changes every period – so learning plays no role. Learning is also crucial in Angeletos, Hellwig and Pavan [2] but they do not consider delay options. In Steiner [25,26], the behavior of outside options is key for coordination and in Chassang [8], exit options might restore equilibrium multiplicity.

As in this paper, Acemoglu and Jackson [1] and Araujo and Guimaraes [3] consider intertemporal coordination problems where an agent's payoff does not depend on the action of other agents in the current period. Lastly, our paper is also related to work providing alternative explanations for why agents inefficiently delay exerting effort, either because of moral hazard (Bonatti and Horner [5]) or because their preferences are time inconsistent (O'Donoghue and Rabin [22]).

2. Model

2.1. Environment

Time is discrete and the discount factor is $\beta \in (0, 1)$. The economy is populated by two agents, labeled active and passive. In every period, the active agent chooses between effort (e) and no effort (n), while the passive agent does not make any decision. If the active agent chooses effort, she incurs a cost c and becomes a passive agent, and the passive agent receives a benefit b and is replaced by a newly born active agent. If the active agent chooses no effort, she continues as an active agent and the passive agent continues as a passive agent.⁵

Payoff-relevant state $z_t \in \mathbb{R}$ evolves according to a random process $z_t = z_{t-1} + \Delta z_t$ where $\Delta z_t = \mu \xi_t$ and ξ_t is drawn from a continuous probability distribution that is independent of z and

² Examples include Frankel and Pauzner [10], Guimaraes [12] and He and Xiong [14].

³ For a survey of the literature on global games, see Morris and Shin [21].

⁴ For example, in Dasgupta [9] and Kovac and Steiner [16], delay options enhance coordination as long as payoffs at the last stage of the game do not depend on early actions.

⁵ In an online appendix, we extend the model to consider different survival probabilities for passive and active agents.

t with probability density $g(\xi)$; $E(\xi) = 0$ and $\text{var}(\xi) > 0$. We further assume that $g(\xi_1) \leq g(\xi_2)$ for all $|\xi_1| > |\xi_2|$.⁶ The probability density and the cumulative distribution of Δz_t are denoted by $f(\Delta z)$ and $F(\Delta z)$.

The benefit b is constant across states, but the effort cost c depends on z_t in the following way: there exists z^0 such that $c(z) = 0$ for $z \leq z^0$, and $c(z)$ is increasing, weakly convex and twice differentiable. This implies that there exists $z_H(\beta)$ such that $c(z_H(\beta)) = \beta b$. Finally, there exists $z^L < z^0$ such that, for all $z < z^L$, if the economy is in state z , the passive agent receives the benefit b regardless of the behavior of the active agent.⁷

2.2. Equilibrium

A symmetric (Nash) equilibrium in cut-off strategies is characterized by a state z^* such that all active agents exert effort if and only if $z < z^*$.⁸ In a symmetric equilibrium in cut-off strategies, an active agent believes all future active agents will follow a cut-off rule. Hence the payoff of an agent does not depend on past states of the economy or on the time t .⁹ Thus there is no further loss in generality in considering that agents' decisions depend only on the current state of the economy, so a strategy of an active agent is given by a mapping from the set of states z to the set of actions (effort and no effort).

Proposition 1 shows sufficient conditions for the existence and uniqueness of a symmetric equilibrium in cut-off strategies (generically). The proof is in Section A.1.

Proposition 1. *There exists a unique symmetric equilibrium in cut-off strategies. Agents exert effort if and only if $z < z^*$, where z^* is the state in which an agent is indifferent between exerting and postponing effort.*

2.3. No delay options

Consider the hypothetical scenario where an active agent can only exert effort in the first period of her life and can only receive the benefit one period after she has made effort. In this case, the agent is essentially playing a one-shot game with her future partner. At the threshold z^* , if an agent exerts effort, she obtains $-c(z^*) + F(0)\beta b$, while if she does not exert effort, she obtains 0. This implies that z^* solves $c(z^*) = F(0)\beta b$. If $F(0) = 1/2$, effort is exerted if and only if it is the risk dominant action of the underlying one-shot game. Indeed, effort yields $\beta b - c(z)$ if the following active agent chooses effort, while no effort saves $c(z)$ if the following active agent

⁶ This assumption is used to prove that the best response to the equilibrium strategy profile is a cut-off strategy, as it ensures that the difference between the value of exerting and not-exerting effort is monotonic in z . It is not used in the characterization of the equilibrium threshold.

⁷ The assumption that $\text{var}(\Delta z) > 0$ plays an important role in the model. In case $\text{var}(\Delta z) = 0$, if $z \in [z_L, z_H(\beta)]$, there are multiple equilibria since $\beta b \geq c(z)$. There exists a no-coordination equilibrium, in which active agents never exert effort, and a coordination equilibrium in which active agents always exert effort. There also exists an unstable mixed strategy equilibrium in which active agents are indifferent and exert effort with probability $\frac{c(z)}{\beta b - c(z)} \frac{1-\beta}{\beta}$ (as long as this probability is smaller than 1). In case $z \notin [z_L, z_H(\beta)]$, one of the actions is strictly dominant.

⁸ By restricting attention to symmetric equilibria in cut-off strategies, we are not imposing that an active agent has to follow a cut-off rule, we allow for (and rule out) deviations where an active agent chooses a strategy that is not of a cut-off type.

⁹ Here it is important that the process for Δz is independent of t and past realizations of Δz .

chooses no effort. Effort is the risk dominant action if $\beta b - c(z) > c(z)$, which holds if and only if $c(z) > c(z^*) = \beta b/2$. For a general distribution for Δz , effort is exerted if it is $F(0)$ -dominant.

3. The strategic irrelevance of delay options

The possibility of delaying effort affects agents' decisions in two ways. First, there is a strategic component: an agent will consider how delay options will affect the behavior of the following agents when deciding on whether to exert effort or wait. Second, there is a standard real-option effect since waiting might allow for a lower effort cost. In this section, we focus on the first effect by considering the case with vanishing shocks. Proposition 2 shows that as the variance of the process for Δz approaches 0, the equilibrium outcome coincides with the hypothetical scenario with no delay options.

Proposition 2. *As $\mu \rightarrow 0^+$,*

$$c(z^*) = F(0)\beta b. \tag{1}$$

A detailed proof is in Section A.2. Below we offer an intuitive explanation, using a simplified version of the model where the real option effect is absent: the cost for an agent that becomes active when the economy is in state \tilde{z} is $c(\tilde{z})$ in the current or in any future period. This simplified version captures all the relevant features of our environment in the case of vanishing shocks.

Consider the problem of a young agent at t_0 in a pivotal circumstance, $z = z^*$. Let t_1 be the first subsequent period such that the economy is at $z < z^*$ and t_2 be the second subsequent period such that the economy is at $z < z^*$. An agent that chooses effort at z^* incurs the cost $c(z^*)$ at t_0 and receives benefit b at t_1 . Let $\tau_1 = t_1 - t_0$ and let $\tau_2 = t_2 - t_1$. Exerting effort implies a cost $c(z^*)$ at t_0 and a benefit b at t_1 , hence the expected payoff is:

$$V_e(z^*) = -c(z^*) + E[\beta^{\tau_1}]b. \tag{2}$$

Delaying effort implies a cost $c(z^*)$ at t_1 and a benefit b at t_2 , hence the expected payoff is:

$$V_n(z^*) = -E[\beta^{\tau_1}]c(z^*) + E[\beta^{\tau_1+\tau_2}]b. \tag{3}$$

The agent is indifferent when $V_e(z^*) = V_n(z^*)$, so:

$$-c(z^*) + E[\beta^{\tau_1}]b = -E[\beta^{\tau_1}]c(z^*) + E[\beta^{\tau_1+\tau_2}]b. \tag{4}$$

The expression above is fairly complicated, as it includes all possible trajectories of the state z , starting at z^* . However, it turns out that, when evaluating the difference between $V_e(z^*)$ and $V_n(z^*)$, we only need to consider the set of evolutions of the state where a negative Δz immediately follows the decision to exert effort. Such evolutions, which happen with probability $F(0)$, provide the agent with the benefit b right after he exerts effort, thus allowing us to write (4) as

$$-c(z^*) + F(0)\beta b = E[\beta^{\tau_1}](-c(z^*) + F(0)\beta b).$$

It is as if the agent is deciding between $-c(z^*) + F(0)\beta b$ now or later and indifference will only occur when $c(z^*) = F(0)\beta b$.

The key result that delivers a simple expression for the cut-off state z^* is the fact that we can discard evolutions of the state where a positive Δz immediately follows effort. The intuition for this result runs as follows. Suppose an agent is at z^* in period $t = 0$ and can either exert effort or wait until the economy is at the left of z^* . Denote by W the set of future evolutions of the state

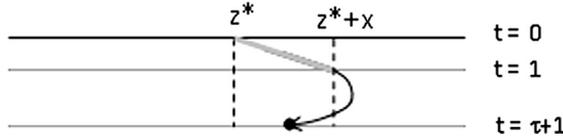


Fig. 1. Effort at z^* , then a positive Δz .

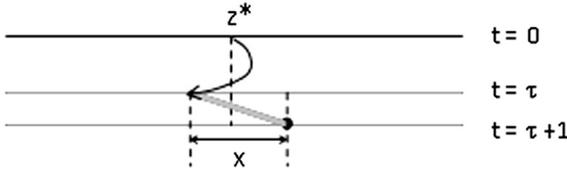


Fig. 2. No effort at z^* , positive Δz after effort is exerted.

where a positive Δz is drawn right after effort is made (at $t = 1$ in case of effort at $t = 0$ and right after the economy gets to the left of z^* in case of no effort at $t = 0$). Restricting our attention to paths in W , the expected time between z^* and the time the agent gets the benefit is exactly the same in both cases. That is better explained with the aid of pictures. Consider the agent exerts effort in $t = 0$ and the following realization of Δz is $x > 0$. Fig. 1 illustrates this case: Let $\tau \in \mathbb{Z}_+$ be the number of periods until the economy is at the left of $z^* + x$ after $t = 1$. The benefit will not be received before $t = \tau + 1$ and at $t = \tau + 1$, the economy will be at \hat{z} for some $\hat{z} < z^* + x$.

Now let us consider the agent does not exert effort at $t = 0$ and let's rearrange the path above, moving the first realization of Δz to the end of the sequence of $\tau + 1$ realizations of Δz , as in Fig. 2. That means the economy will be at the left of z^* for the first time at $t = \tau$. At this point, the agent will exert effort. The first realization of Δz from the previous example follows (and it is positive). The important point is that the benefit will not be received before $t = \tau + 1$ and at $t = \tau + 1$, the economy will be at \hat{z} for the same value of \hat{z} from the previous example.

The assumption that Δz 's are serially uncorrelated implies the probability of a sequence in Fig. 1 is equal to the probability of its counterpart in Fig. 2. Hence attributing a value of 0 to all paths in W does not affect the indifference condition.

If the distribution for Δz is symmetric, $F(0) = 1/2$ and the risk dominant action is played, as in the dynamic models of Matsui and Matsuyama [19] and Burdzy, Frankel and Pauzner [6]. However, the environment is very different: while in those models agents play a repeated 2×2 game and get opportunities to revise their behavior according to a Poisson process, here agents effectively choose when to exert effort. The risk-dominant action is selected because in a pivotal condition, the possibility of waiting longer for the benefit is equal to the value of postponing effort. This consideration is absent from those papers.

4. The real option effect

In this section, we study the case when μ is bounded away from 0. The expected payoff from exerting effort is the same as in (2) while the expected payoff from delaying effort becomes

$$V_n(z^*) = -E[\beta^{\tau_1}]c(z_{t_1}) + E[\beta^{\tau_1 + \tau_2}]b.$$

where $c(z_{t_1})$ is the expected cost of exerting effort conditional on t_1 . As before, we only need to consider the set of evolutions of the state where a negative Δz immediately follows the decision to exert effort. This implies that we can write $V_e(z^*) = V_n(z^*)$ as

$$(1 - E[\beta^{\tau_1}])(-c(z^*) + F(0)\beta b) = E[\beta^{\tau_1}](c(z^*) - c(z_{t_1})) \quad (5)$$

Since $z^* > z_{t_1}$, the left hand side of (5) must be positive, hence the cut-off state z^* must be smaller than in the case of vanishing shocks. Intuitively, effort becomes more difficult because agents take into account that waiting might allow for a lower effort cost. A more complete characterization of the equilibrium threshold can be found in Section A.2.

It can be shown that an increase in the variance of the process for Δz harms coordination because postponing effort allows for a lower effort cost in the future. Hence the threshold given by (1) is an upper bound for the equilibrium obtained with the same parameters for any distribution of Δz .

5. Application

We can cast our model as an OLG model of money (see, for instance, Sargent [24]) augmented by delay options. There exists a storable and indivisible asset, labeled money, initially given to the passive agent. In every period, an agent is passive if she holds money, and she is active otherwise. Passive agents make no choice while active agents choose between exerting effort in exchange for money and not exerting effort. Exerting effort entails a cost $c(z)$ and provides a benefit b for the passive agent.¹⁰ This environment maps into our model. The difference with the standard OLG model of money is that in the latter, an active (young) agent only has one opportunity to exert effort in exchange for money, and a passive (old) agent who exerted effort only has one opportunity to reap the benefits of holding money.

Absent delay options, there exists a unique equilibrium in which money circulates if and only if to produce in exchange for money is the risk-dominant action of the underlying one-shot game between the current active agent and the future active agent. One may naturally expect a different outcome in the presence of delay options. In one direction, the possibility of delaying production in exchange for money may reduce the acceptability of money. In another direction, the possibility of having more opportunities to exchange money for a desirable good may increase its acceptability. It turns out that, following Proposition 2, these incentives exactly offset each other when frictions vanish: the risk-dominant action of the underlying one-shot game is always selected. The message is that, when considering equilibrium selection in an OLG model of money, one should not be too bothered with the fact that, by assumption, this model does not allow delay options.

Our model can also be used to study instances where an asset holds both a fundamental value and a liquidity value. These instances became particularly relevant in the wake of the recent financial crisis.¹¹ Building in the discussion above, assume that, after exerting effort in exchange for the asset, the active agent can now choose between leaving the economy with the asset, obtaining a one-time utility $u(z)$; and staying in the economy, in which case she does not obtain any intrinsic utility from the asset but becomes a passive agent. If we redefine the (net) cost of effort as $c(z) - u(z)$, this scenario maps into our environment. An interesting exercise consists in interpreting an increase in the variance of Δz as capturing an increase in uncertainty about the

¹⁰ $c(z)$ and z follow the assumptions laid out in Section 2.

¹¹ For instance, Gorton and Ordóñez [11] consider an OLG setting in which an asset (land) holds both a fundamental value and a liquidity value, as it can be used as collateral in transactions.

fundamental value of the asset, measured by $u(z)$.¹² In this case, the region of parameters where it is strictly optimal to produce in exchange for the asset decreases. This is due to a real options effect that affects how agents coordinate.¹³

6. Conclusion

The overlapping generation model provides a tractable setting for studying asset trade, but one unappealing feature of this framework is that agents have only one chance to exert effort. This paper augments the standard OLG model in order to include these options and shows that from a strategic point of view, delay options do not affect coordination. The model also adds to the collection of games with strategic complementarities where the risk-dominant action is chosen.

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Appendix A. Proofs

A.1. Proof of Proposition 1

Let $V_e(z_1, z_3)$ be the expected payoff from exerting effort in state z_1 if future active agents are following a cut-off rule at state z_3 . We have

$$V_e(z_1, z_3) = -c(z_1) + \beta \int_{-\infty}^{\infty} f(w) V_1(z_1 + w, z_3) dw, \quad (\text{A.1})$$

where $V_1(z_1 + w, z_3)$ is the expected payoff of being a passive agent in state $z_1 + w$, given that future active agents are following a cut-off rule at state z_3 . In turn, let $V_n(z_1, z_2, z_3)$ be the expected payoff from not exerting effort in state z_1 , and exerting effort in the following periods if and only if $z < z_2$, given that future active agents are following a cut-off rule at state z_3 . Then,

$$V_n(z_1, z_2, z_3) = \int_{z_1 - z_2}^{\infty} \Gamma_x \left(-c(z_1 - x) + \beta \int_{-\infty}^{\infty} f(w) V_1(z_1 - x + w, z_3) dw \right) dx. \quad (\text{A.2})$$

where

¹² Gorton and Ordóñez [11] are particularly interested on how asymmetric information about the fundamental value of land affects its liquidity. Here we are interested on how common uncertainty about the future value of the asset affects its liquidity.

¹³ Martin and Ventura [17] is another recent model of asset trading that employs an OLG framework.

$$\Gamma_x \equiv \sum_{t=1}^{\infty} \beta^t \psi(z_1 - x, z_1, z_2, t).$$

and $\psi(z_1 - x, z_1, z_2, t)$ denotes the probability density that the state of the economy is $z_1 - x$ in period $s + t$, conditional on z_1 being the state of the economy in period s , and conditional on the economy not being in any state $z \leq z_2$ in periods $\{s + 1, \dots, s + t - 1\}$.

Lemma 1. $\Delta v(z) \equiv V_e(z, z) - V_n(z, z, z)$ is strictly decreasing in $z \leq z^L$, constant in $z \in [z^L, z^0]$, and strictly decreasing in $z \geq z^0$.

Proof. We start with $z \geq z^L$. $\Delta v(z)$ can be written as

$$\begin{aligned} & -c(z) + \beta \int_{-\infty}^{\infty} f(w) V_1(z + w, z) dw \\ & - \int_0^{\infty} \Gamma_x \left\{ -c(z - x) + \beta \int_{-\infty}^{\infty} f(w) V_1(z - x + w, z) dw \right\} dx, \end{aligned}$$

Since $V_1(z + w, z) = V_1(z^L + w, z^L)$ and $V_1(z - x + w, z) = V_1(z^L - x + w, z^L)$, the benefit part of the payoff does not depend on z . This implies that

$$\frac{\partial \Delta v(z)}{\partial z} = -c'(z) + \int_0^{\infty} \Gamma_x c'(z - x) dx \leq -c'(z) \left(1 - \int_0^{\infty} \Gamma_x dx \right),$$

where the inequality comes from the convexity of $c(z)$. If $z \leq z^0$, $c'(z) = 0$ and $\Delta v(z)$ is constant. If $z > z^0$, $c'(z) > 0$, hence $\Delta v(z)$ is strictly decreasing in z .

Consider now $z < z^L$. The payoff from exerting effort at z is

$$V_e(z, z) = \beta \int_{-\infty}^{\infty} f(w) V_1(z + w, z^L) dw,$$

where we used the fact that $V_1(z, z) = V_1(z, z^L)$ when $z < z^L$. Hence

$$\frac{\partial V_e(z, z)}{\partial z} = \beta \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z + w, z^L)}{\partial z} dw.$$

The payoff from not exerting effort at z is

$$V_n(z, z, z) = \int_0^{\infty} \Gamma_x \beta \int_{-\infty}^{\infty} f(w) V_1(z - x + w, z^L) dw dx.$$

Since x is defined as the distance between the current state $z - x$ and the state z , a change in z does not affect Γ_x . This implies that

$$\frac{\partial V_n(z, z, z)}{\partial z} = \int_0^{\infty} \Gamma_x \beta \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z - x + w, z^L)}{\partial z} dw dx.$$

Therefore

$$\frac{\partial \Delta v(z)}{\partial z} \frac{1}{\beta} = \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z+w, z^L)}{\partial z} dw - \int_0^{\infty} \Gamma_x \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z-x+w, z^L)}{\partial z} dwdx.$$

Since $\int_0^{\infty} \Gamma_x dx < 1$, we have

$$\begin{aligned} \frac{\partial \Delta v(z)}{\partial z} \frac{1}{\beta} &< \int_0^{\infty} \Gamma_x \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z+w, z^L)}{\partial z} dwdx \\ &\quad - \int_0^{\infty} \Gamma_x \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z-x+w, z^L)}{\partial z} dwdx. \end{aligned}$$

After a change in variables, we can rewrite the above inequality as

$$\begin{aligned} \frac{\partial \Delta v(z)}{\partial z} \frac{1}{\beta} &< \int_0^{\infty} \Gamma_x \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z+w, z^L)}{\partial z} dwdx \\ &\quad - \int_0^{\infty} \Gamma_x \int_{-\infty}^{\infty} f(\tilde{w}+x) \frac{\partial V_1(z+\tilde{w}, z^L)}{\partial z} d\tilde{w}dx. \end{aligned}$$

Now since $\frac{\partial V_1(z, z^L)}{\partial z} = 0$ for $z < z^L$, $\frac{\partial V_1(z, z^L)}{\partial z} < 0$ for $z \geq z^L$, and $f(w) > f(w+x)$ for all $x > 0$ and $w > 0$, we obtain

$$\frac{\partial \Delta v(z)}{\partial z} \frac{1}{\beta} < \int_0^{\infty} \Gamma_x \int_{z^L-z}^{\infty} [f(w) - f(w+x)] \frac{\partial V_1(z+w, z^L)}{\partial z} dwdx < 0.$$

This concludes our proof. \square

Note that $\Delta v(z)$ converges to a positive number when z goes to $-\infty$, and it converges to a negative number when z goes to ∞ . Thus, with the exception of a measure zero set of parameters implying $\Delta v(z) = 0$ for all $z \in [z^L, z^0]$, there exists a unique $z \in \mathbb{R}$ such that $\Delta v(z) = 0$. This completes the first step of the proof. Henceforth, we let z^* denote the unique value of z such that $\Delta v(z) \equiv V_e(z, z) - V_n(z, z, z) = 0$.

Lemma 2. $\Delta V^*(z, z^*) \equiv V_e(z, z^*) - V_n^*(z, z^*) < 0$ if $z > z^*$.

Proof. Since $V_n^*(z, z^*) \geq V_n(z, z^*, z^*)$, it is enough to show that $V_e(z, z^*) - V_n(z, z^*, z^*) < 0$.

Fix some period s and consider the effects on $V_e(z, z^*)$ of an arbitrarily small increase in z . First, an arbitrarily small increase in z implies an increase $c'(z)$ in the cost of exerting effort. Second, an arbitrarily small increase in z implies a decrease in the continuation payoff of exerting effort. This decrease is given by the discounted probability density associated with the event that z^* is the first state $z \leq z^*$ reached after period s , multiplied by the difference between obtaining the benefit b at z^* and not obtaining the benefit b at z^* . Indeed, it is only when z^* is the first state $z \leq z^*$ reached that the continuation payoff of exerting effort at z is different from the

continuation payoff of exerting effort at a state $z + dz$, where dz is arbitrarily close to zero. Formally, we have

$$\frac{\partial V_e(z, z^*)}{\partial z} = -c'(z) - \sum_{t=1}^{\infty} \beta^t \psi(z^*, z, z^*, t) [b - \widehat{V}_e(z^*)],$$

where $\widehat{V}_e(z^*)$ is the payoff of being a passive agent at z^* under the assumption that effort is only exerted by active agents in states to the left of z^* .

Now fix some period s and consider the effects on $V_n(z, z^*, z^*)$ of an arbitrarily small increase in z . First, if dz is arbitrarily small and the agent is currently at state $z > z^*$, the expected payoff of exerting effort only at states $z' < z^*$ coincides with the expected payoff of exerting effort only at states $z' < z^* + dz$. This implies that, for dz arbitrarily small, $V_n(z, z^*, z^*) = V_n(z + dz, z^* + dz, z^*)$. Now, since the process for Δz does not depend on z , this also implies that the first period in which effort is exerted is the same for an agent who is currently in state z and follows a cut-off rule at z^* and an agent who is currently in state $z + dz$ and follows a cut-off rule at $z^* + dz$. Thus, as in the case of $V_e(z, z^*)$, the only difference in payoffs associated with an arbitrarily small increase in z comes from differences in the marginal cost of effort and differences in continuation payoffs which arise if the state z^* is the first state $z' \leq z^*$ that is reached after the agent has made effort. Formally, we have that:

$$\begin{aligned} \frac{\partial V_n(z, z^*, z^*)}{\partial z} = & \int_0^{\infty} \sum_{t=1}^{\infty} \beta^t \psi(z^* - x, z, z^*, t) \left\{ -c'(z^* - x) \right. \\ & \left. - \sum_{k=1}^{\infty} \beta^k \psi(z^*, z^* - x, z^*, k) [b - \widehat{V}_e(z^*)] \right\} dx. \end{aligned}$$

Now, note that

$$\sum_{k=1}^{\infty} \beta^k \psi(z^*, z^* - x, z^*, k) \leq 1,$$

so that a lower bound for $\frac{\partial V_n(z, z^*, z^*)}{\partial z}$ is given by

$$- \int_0^{\infty} \sum_{t=1}^{\infty} \beta^t \psi(z^* - x, z, z^*, t) c'(z^* - x) dx - \int_0^{\infty} \sum_{t=1}^{\infty} \beta^t \psi(z^* - x, z, z^*, t) [b - \widehat{V}_e(z^*)] dx.$$

Moreover, the assumption that the density of Δz is weakly decreasing in Δz for $\Delta z > 0$ and weakly increasing in Δz for $\Delta z < 0$ implies that, for all $x > 0$,

$$\sum_{t=1}^{\infty} \beta^t \psi(z^*, z, z^*, t) \geq \sum_{t=1}^{\infty} \beta^t \psi(z^* - x, z, z^*, t).$$

Combining this fact with the convexity of $c(z)$, we obtain that a lower bound for $\frac{\partial V_n(z, z^*, z^*)}{\partial z}$ is given by

$$-c'(z) \int_0^{\infty} \sum_{t=1}^{\infty} \beta^t \psi(z^* - x, z, z^*, t) dx - \sum_{t=1}^{\infty} \beta^t \psi(z^*, z, z^*, t) [b - \widehat{V}_e(z^*)].$$

Hence an upper bound for $V_e(z, z^*) - V_n(z, z^*, z^*)$ is given by

$$-c'(z) \left[1 - \int_0^\infty \sum_{t=1}^\infty \beta^t \psi(z^* - x, z, z^*, t) dx \right] < 0,$$

which completes the proof. Note that in the analysis above we implicitly assumed that $z^* \geq z^L$. The proof in the case where $z^* < z^L$ is essentially the same. The only difference is that, when considering the impact of an arbitrarily small increase in z on the benefit of exerting effort, we need to replace the state z^* by the state z^L . Indeed, when $z^* < z^L$, the only difference in payoffs associated with an arbitrarily small increase in z comes from differences in the marginal cost of effort and differences in continuation payoffs which arise if the state z^L is the first state $z \leq z^L$ that is reached after the agent has made effort. \square

We now show that if $\Delta V^*(z, z^*) \equiv V_e(z, z^*) - V_n^*(z, z^*) < 0$, then $\Delta V^*(z, z^*)$ is strictly decreasing in z . This implies that $\Delta V^*(z, z^*)$ crosses the zero line at most once. Moreover, since $\Delta V^*(z, z^*) > 0$ for z sufficiently small, together with Lemma 2, this result also implies that the best reply to other agents' following a cut-off rule at z^* is to also follow a cut-off rule at some state $z \leq z^*$.

Lemma 3. *Let $z < z^*$ and assume that $\Delta V^*(z, z^*) \equiv V_e(z, z^*) - V_n^*(z, z^*) < 0$. Then, $\Delta V^*(z, z^*)$ is strictly decreasing in z .*

Proof. Consider an agent at state $z < z^*$ in some period s . If she exerts effort her payoff is

$$V_e(z, z^*) = -c(z) + \beta \int_{-\infty}^\infty f(w) V_1(z + w, z^*) dw,$$

which implies

$$\frac{\partial V_e(z, z^*)}{\partial z} = -c'(z) + \beta \int_{-\infty}^\infty f(w) \frac{\partial V_1(z + w, z^*)}{\partial z} dw.$$

We now look at the agent's optimal payoff if she chooses not to exert effort at z . Let ϑ be the set of states in which it is optimal to exert effort. Let $\vartheta_1 = \vartheta \cap (-\infty, z)$ and $\vartheta_2 = \vartheta \cap (z, \infty)$. Define $\phi_1(z_1, z_2, t)$ as the probability density associated with reaching the state z_1 in period $s + t$, conditional on z_2 being the state of the economy in period s , and conditional on the economy not reaching any state in the set ϑ_1 in periods $\{s + 1, \dots, s + t - 1\}$. Also define $\phi_2(z_1, z_2, z_3, t)$ as the probability density associated with reaching state $z_3 \in \vartheta_2$ at least once in periods $\{s + 1, \dots, s + t - 1\}$, conditional on z_2 being the state of the economy in period s and z_1 being the state of the economy in period $s + t$. Finally, let

$$\Phi_1(z_1, z_2) = \sum_{t=1}^\infty \beta^t \phi_1(z_1, z_2, t)$$

and

$$\Phi_2(z_1, z_2, z_3) = \sum_{\tau=1}^{t-1} \beta^\tau \phi_2(z_1, z_2, z_3, t).$$

The payoff $V_n^*(z, z^*)$ of not exerting effort at z is then given by (where we used the fact that, for every $z' \in \vartheta_1$, there exists $x \geq 0$ such that $z' = z - x$)

$$\int_{\vartheta_1} \Phi_1(z', z) \left\{ -c(z - x) + \beta \int_{-\infty}^{\infty} f(w) V_1(z - x + w, z^*) dw \right. \\ \left. + \int_{\vartheta_2} \Phi_2(z', z, y) [\widehat{V}_e(\cdot) - \widehat{V}_n^*(\cdot)] dy \right\} dz'.$$

To understand the expression above, consider the hypothetical case in which $\vartheta_2 = \emptyset$, i.e., effort is never optimal to the right of state z . In this case, the second integral in the term inside brackets is zero, and the expression is similar to the one in the case where the agent is following a cut-off rule at z . Thus, the second integral in the term inside brackets should be interpreted as the additional payoff an agent receives when it is also optimal to choose effort in states $z \in \vartheta_2$. Even though the exact expression for $\widehat{V}_e(\cdot) - \widehat{V}_n^*(\cdot)$ is complicated, we know that it is positive by the definition of ϑ_2 .

Consider now an arbitrarily small increase in z . If we add the same increase dz to each state in the set ϑ_1 , we obtain that, whenever an agent who is currently in state z chooses effort upon reaching some state $z' \in \vartheta_1$, then an agent who is currently at state $z + dz$ will also choose effort. Note though that, while the behavior of the former agent is optimal (by the definition of ϑ_1), the same is not true of the latter agent. This implies that a lower bound on the increase in the payoff $V_n^*(z, z^*)$ associated with an arbitrarily small increase dz in z is given by

$$\int_{\vartheta_1} \Phi_1(z', z) \left\{ -c'(z - x) + \beta \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z - x + w, z^*)}{\partial z} dw \right. \\ \left. + \int_{\vartheta_2} \frac{\partial \Phi_2(z', z, y)}{\partial z} [\widehat{V}_e(\cdot) - \widehat{V}_n^*(\cdot)] dy \right\} dz'.$$

Now, since an agent who is currently at state $z + dz$ is more likely to eventually reach a state in $\vartheta_2 = \vartheta \cap (z, \infty)$ than an agent who is currently at state z , we know that $\frac{\partial \Phi_2(z', z, y)}{\partial z} > 0$. This implies that a lower bound on the expression above is given by

$$\int_{\vartheta_1} \Phi_1(z', z) \left\{ -c'(z - x) + \beta \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z - x + w, z^*)}{\partial z} dw \right\} dz'.$$

Thus, an upper bound on $\frac{\partial V_e(z, z^*)}{\partial z} - \frac{\partial V_n^*(z, z^*)}{\partial z}$ is

$$-c'(z) + \beta \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z + w, z^*)}{\partial z} dw \\ - \int_{\vartheta_1} \Phi_1(z', z) \left\{ -c'(z - x) + \beta \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z - x + w, z^*)}{\partial z} dw \right\} dz'.$$

Since $c(z)$ is convex and $\int_{\vartheta_1} \Phi_1(z', z) dz' < 1$, an upper bound on the expression above is

$$\beta \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z+w, z^*)}{\partial z} dw - \int_{\vartheta_1} \Phi_1(z', z) \beta \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z-x+w, z^*)}{\partial z} dw dz'$$

Moreover, since $\frac{\partial V_1(z+w, z^*)}{\partial z} \leq 0$ (and using $\int_{\vartheta_1} \Phi_1(z', z) dz' < 1$), an upper bound on the expression above is

$$\beta \int_{\vartheta_1} \Phi_1(z', z) \left[\int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z+w, z^*)}{\partial z} dw - \int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z-x+w, z^*)}{\partial z} dw \right] dz'$$

Finally, a change in variables allows us to rewrite the above expression as

$$\beta \int_{\vartheta_1} \Phi_1(z', z) \left[\int_{-\infty}^{\infty} f(w) \frac{\partial V_1(z+w, z^*)}{\partial z} dw - \int_{-\infty}^{\infty} f(w+x) \frac{\partial V_1(z+w, z^*)}{\partial z} dw \right] dz'$$

We know that $\frac{\partial V_1(z+w, z^*)}{\partial z} = 0$ whenever $z+w < z^*$, i.e., $w < z^* - z$. Thus, we can rewrite the above expression as

$$\beta \int_{\vartheta_1} \Phi_1(z', z) \int_{z^*-z}^{\infty} [f(w) - f(w+x)] \frac{\partial V_1(z+w, z^*)}{\partial z} dw dz'$$

Finally, since $w > 0$ and $x \geq 0$, we have $f(w) > f(w+x)$ and the above expression is negative. □

We have thus shown that the best reply to other agents following a cut-off rule at z^* is to also follow a cut-off rule at some state $z \leq z^*$. Lemma 4 proves that $z < z^*$ cannot be a cut-off, which implies that effort is exerted in all states $z < z^*$.

Lemma 4. For all $z < z^*$, $\Delta V^*(z, z^*) \equiv V_e(z, z^*) - V_n^*(z, z^*) > 0$.

Proof. Consider the decision of an agent at $z < z^*$. The payoff from exerting effort is given by

$$V_e(z, z^*) = -c(z) + \beta \int_{-\infty}^{\infty} f(w) V_1(z+w, z^*) dw,$$

while the payoff from not exerting effort and following a cut-off rule at z is

$$V_n(z, z, z^*) = \int_0^{\infty} \Gamma_x \left[-c(z-x) + \beta \int_{-\infty}^{\infty} f(w) V_1(z-x+w, z^*) dw dx \right]$$

Now, note that $V_e(z, z^*) - V_n(z, z, z^*) - \Delta v(z^*)$ is given by

$$c(z^*) - c(z) - \int_0^{\infty} \Gamma_x [c(z^* - x) - c(z - x)] dx$$

$$\begin{aligned}
 & + \beta \int_{-\infty}^{\infty} f(w) [V_1(z + w, z^*) - V_1(z^* + w, z^*)] dw \\
 & - \beta \int_0^{\infty} \Gamma_x \int_{-\infty}^{\infty} f(w) [V_1(z - x + w, z^*) - V_1(z^* - x + w, z^*)] dw dx.
 \end{aligned}$$

We will show that this expression is positive. Since $\Delta v(z^*) = 0$, this implies that $V_e(z, z^*) - V_n(z, z, z^*) > 0$. Thus, z is not an optimal response to all other agents choosing a cut-off at z^* .

First, consider the first line in the expression above. For any $x \geq 0$,

$$c(z^*) - c(z) \geq c(z^* - x) - c(z - x) \geq 0,$$

which comes from $z^* > z$ and c being convex. Since the integral of Γ_x in x is smaller than one, the first line in the expression above must be positive. Now consider the second and third lines. We have

$$\begin{aligned}
 & \beta \int_{-\infty}^{\infty} f(w) [V_1(z + w, z^*) - V_1(z^* + w, z^*)] dw \\
 & - \beta \int_0^{\infty} \Gamma_x \int_{-\infty}^{\infty} f(w) (V_1(z - x + w, z^*) - V_1(z^* - x + w, z^*)) dw dx
 \end{aligned}$$

Since the integral of Γ_x in x is smaller than one and the first integral is positive ($V_1(z, z^*)$ is decreasing in z), a lower bound on the expression above is

$$\begin{aligned}
 & \beta \int_0^{\infty} \Gamma_x \int_{-\infty}^{\infty} f(w) \left\{ - \left[\begin{aligned} & [V_1(z + w, z^*) - V_1(z^* + w, z^*)] \\ & - [V_1(z - x + w, z^*) - V_1(z^* - x + w, z^*)] \end{aligned} \right] \right\} dw dx \\
 & = \beta \int_0^{\infty} \Gamma_x \int_0^{\infty} f(w) \left\{ - \left[\begin{aligned} & [V_1(z + w, z^*) - V_1(z^* + w, z^*)] \\ & - [V_1(z - x + w, z^*) - V_1(z^* - x + w, z^*)] \end{aligned} \right] \right\} dw dx
 \end{aligned}$$

where the lower limit of the inner integrals were changed from $-\infty$ to 0. This can be done because $V_1(z) = b$ for $z \leq z^*$. A change in variables allows us to rewrite the above expression as

$$\beta \int_0^{\infty} \Gamma_x \int_0^{\infty} [f(w) - f(w + x)] [V_1(z + w, z^*) - V_1(z^* + w, z^*)] dw dx.$$

This last expression is non-negative since $f(w) \geq f(w + x)$ for w and x nonnegative. \square

We have thus shown that there exists a unique equilibrium in symmetric strategies. In this equilibrium, agents exert effort if and only if $z < z^*$, where z^* solves

$$\begin{aligned}
 & -c(z^*) + \beta \int_{-\infty}^{\infty} f(w) V_1(z^* + w, z^*) dw \\
 & = \int_0^{\infty} \Gamma_x \left[-c(z^* - x) + \beta \int_{-\infty}^{\infty} f(w) V_1(z^* - x + w, z^*) dw dx \right].
 \end{aligned}$$

A.2. Proof of Proposition 2

It is convenient to rewrite the expression in (2) so that the expected payoff of exerting effort at z^* in period s , conditional on all future active agents exerting effort at z^* is given by:

$$V_e(z^*, z^*) = -c(z^*) + F(0)\beta b + \left[\int_0^\infty f(x)\Omega_x dx \right] \beta b, \tag{A.3}$$

where Ω_x is the sum of probabilities that the economy will be at a state smaller than z^* for the first time in period $s + t$ discounted by β^t , given that the economy is currently in state $z^* + x$. The function Ω_x can be recursively written as:

$$\Omega_x = \int_x^\infty \Gamma_w dw + \int_0^x \Gamma_w \Omega_{x-w} dw, \tag{A.4}$$

where Γ_w is given by

$$\Gamma_w = \sum_{t=1}^\infty \beta^t \phi(w, t)$$

and $\phi(w, t)$ is the probability density that the economy will be in a state smaller than z after t periods but not before and that at time t the economy will be in state $z - w$.

The first integral in (A.4) considers all processes such that the economy is at a state smaller than z^* the first time it is at a state smaller than $z^* + x$. The second integral considers occurrences where the economy reaches a state between z^* and $z^* + x$ before eventually reaching a state smaller than z^* for the first time. It is also convenient to rewrite the expression in (3) so that the expected payoff of not exerting effort at z^* in period s , conditional on all active agents (including the agent himself) following a cut-off rule at state z^* in all future periods is given by

$$V_n(z^*, z^*, z^*) = \int_0^\infty \Gamma_x \left[-c(z^* - x) + \beta b \left(F(x) + \int_0^\infty f(x+w)\Omega_w dw \right) \right] dx. \tag{A.5}$$

Using (A.3) and (A.5), we get that $V_e(z^*, z^*) = V_n(z^*, z^*, z^*)$ when

$$\begin{aligned} & -c(z^*) + F(0)\beta b + \left[\int_0^\infty f(x)\Omega_x dx \right] \beta b \\ &= \int_0^\infty \Gamma_x \left[-c(z^* - x) + \beta b \left(F(x) + \int_0^\infty f(x+w)\Omega_w dw \right) \right] dx \end{aligned} \tag{A.6}$$

Proposition 3 provides a more complete characterization of the equilibrium threshold.

Proposition 3. *The equilibrium threshold z^* satisfies:*

$$-c(z^*) + F(0)\beta b = \int_0^\infty \Gamma_x (-c(z^* - x) + F(0)\beta b) dx \tag{A.7}$$

Proof. First, we show that

$$\int_0^\infty f(x)\Omega_x dx = \int_0^\infty \Gamma_x \left[F(x) - F(0) + \int_0^\infty f(w+x)\Omega_w dw \right] dx \tag{A.8}$$

Using the recursive form of Ω_w , we can write the left-hand side of (A.8) as

$$\int_0^\infty f(x)\Omega_x dx = \int_0^\infty \int_x^\infty f(x)\Gamma_w dw dx + \int_0^\infty \int_0^x f(x)\Gamma_w \Omega_{x-w} dw dx. \tag{A.9}$$

Manipulating the first term on the right-hand side of (A.9), we get:

$$\int_0^\infty \int_x^\infty f(x)\Gamma_w dw dx = \int_0^\infty \int_0^w \Gamma_w f(x) dx dw = \int_0^\infty \Gamma_w [F(w) - F(0)] dw,$$

where the first equality comes from changing the order of variables in the double integral. In turn, manipulating the second term on the right-hand side of (A.9), we get

$$\int_0^\infty \int_x^\infty f(x)\Gamma_w \Omega_{x-w} dw dx = \int_0^\infty \int_w^\infty \Gamma_w f(x)\Omega_{x-w} dx dw = \int_0^\infty \int_0^\infty \Gamma_w f(y+w)\Omega_y dy dw,$$

where the first equality comes from changing the order of variables in the double integral and the second line comes from making $y = x - w$. We can thus rewrite the left-hand side of (A.9) as

$$\int_0^\infty f(x)\Omega_x dx = \int_0^\infty \Gamma_x [F(x) - F(0)] dx + \int_0^\infty \int_0^\infty \Gamma_x f(w+x)\Omega_w dw dx,$$

which yields the expression in (A.8). Using (A.8) and the equilibrium condition in (A.6), we get to (A.7). □

As $\mu \rightarrow 0^+$, $c(z^* - x) \rightarrow c(z^*)$ and the condition in (A.7) can be written as

$$\left(1 - \int_0^\infty \Gamma_x dx \right) [-c(z^*) + F(0)\beta b] = 0$$

which yields $c(z^*) = F(0)\beta b$.

Appendix B. Supplementary material

Supplementary material related to this article can be found online at <http://dx.doi.org/10.1016/j.jet.2015.02.008>.

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